

Fitzwilliam Maths Circle
Topic: Logic

2026-01-10

Exercise 1. Explain why the following argument is logically correct.

1. Everyone loves my baby;
2. My baby loves only me;
3. Therefore, I am my own baby.

Exercise 2. Rewrite each of the following sentences to be of the form “If P , then Q .” Make sure your new wording does not change its meaning.

- (a) A group is cyclic whenever it is of prime order.
- (b) Being differentiable is a sufficient criterion for a function to be continuous.
- (c) A set A has infinitely many elements only if $|A| \geq |\mathbb{N}|$.
- (d) Every polynomial is continuous.

Exercise 3. True or false: The flying panda in this room is riding a centaur.

Hint: Think carefully about vacuous truth and the truth table for implications.

Exercise 4. Negate the following sentences.

- (a) For every prime p , there exists a prime q for which $q > p$.
- (b) If $xy = 0$, then $x = 0$ or $y = 0$.
- (c) If mn is odd, then m is odd and n is odd.
- (d) For every $\varepsilon > 0$ there exists an N such that $n > N$ implies $|a_n - a| < \varepsilon$.

Exercise 5. Translate each of the following to plain English, and then write down whether each statement is true or false.

- (a) $\exists x \in \mathbb{N}$ such that $\forall y \in \mathbb{N}$, we have $x \leq y$.

- (b) $\exists a, b \in \mathbb{N}$ such that $a \neq b$ and $a^b = b^a$.
- (c) $\forall x \in \mathbb{R} \exists y \in \mathbb{N}$ such that $xy \leq 0$.
- (d) $\forall x, y \in \mathbb{R} \exists z \in \mathbb{Q}$ such that $x < z < y$.

Exercise 6. Prove that for all $x, y \in \mathbb{Q}$, there exists some $z \in \mathbb{Q}$ such that $x < z < y$.

Exercise 7. Determine which of the following are true. If it is true, just say so. If it is false, give a counterexample.

- (a) There exists some $n \in \mathbb{N}$ such that $\sqrt{n} \in \mathbb{N}$.
- (b) There exists some $n \in \mathbb{N}$ such that $\sqrt{n} \notin \mathbb{N}$.
- (c) For all $x \in \mathbb{R}$ there exists some $y \in \mathbb{R}$ such that $x^2 = y$.
- (d) For all $x \in \mathbb{R}$ there exists some $y \in \mathbb{R}$ such that $y^2 = x$.
- (e) For all $x \in \mathbb{R}$ there exists some $y \in \mathbb{R}$ such that $y^3 = x$.

Exercise 8. You are investigating murder. The following facts have been established at the scene of the crime.

1. If Colonel Mustard is not guilty, then the crime took place in the library.
2. Either the weapon was the wrench or the crime took place in the billiard room.
3. If the crime took place at midnight, then Colonel Mustard is guilty.
4. Professor Plum is innocent if and only if the weapon was not the wrench.
5. Either Professor Plum or Colonel Mustard is guilty.

Now, you are also an attendee of the Fitzwilliam Maths Circles. As such, you start wondering what further piece of evidence would conclusively determine the killer. For each of the following, explain how that piece of evidence, if established, would determine the killer.

- (a) The crime took place in the billiard room.
- (b) The crime did not take place in the billiard room.
- (c) The crime was committed at noon with a knife.
- (d) The crime took place at midnight in the kitchen.

Source: Problems adapted from *Proofs: A Long-Form Mathematics Textbook* by Jay Cummings.